Bob Jewett

When Spheres Collide

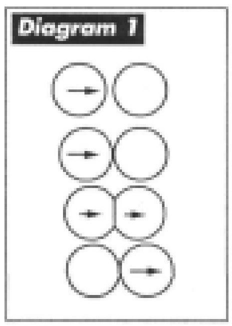
More here than meets the eye.

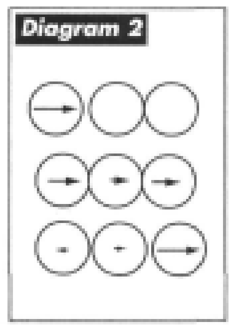
In last month's column, I mentioned in passing that pool balls compress during collisions. This seems to be contrary to what you see on the table: the balls appear to be hard, incompressible spheres. In fact, there must be some "give" to the surface, or they would not behave nearly as perfectly as they do.

In July 1998, I explained that to study a cue stick, you should think of it as small lumps of mass joined by short, stiff springs. Since the naked eye can't see the stick compressing along its length during a shot, you might conclude that the stick was perfectly stiff, did not compress, and delivered its energy through the tip to the ball nearly instantaneously. In fact, what happens is that ball pushes on the tip and compresses it, the tip pushes on and compresses the ferrule, which pushes on the shaft, which then pushes through the joint, into the butt and finally to the back end of the cue. As the ball comes off the tip, all this compression is relaxed, and the energy stored in the compression of both the tip and the stick itself is mostly released into the cue ball. I say "mostly" because both the stick and the tip are not perfectly springy. Some energy is lost into the stick/tip combination, but if there were no springiness at all, the cue ball would have only about 60% of the speed we see.

Do you remember from high school what sound waves are? They are a similar compression of the air by something vibrating. Usually the sound energy doesn't come back, but when it does, it's an echo. You can think of the compression of the stick as echoing off the back end, and in some sense doubling the speed of the cue ball. Like an echo, the compression of the stick moves with the speed of sound. Unlike the echo, the speed in the stick is not the standard speed of sound — lightning a mile away will be heard in five seconds — but is the speed of sound in the wood of the stick.

By now it should be clear that ball-ball collisions aren't as simple as they may seem on the surface. When the cue ball hits an object ball, a sequence like the one for the stick-ball begins. This is shown in **Diagram 1,** with the compression slightly exaggerated. At the first instant of contact, the cue ball is still moving forward but the object ball hasn't started to move yet. Something has to give, and what gives is the spherical shape of the balls. As they move together, flat spots develop on each one as the plastic in the contact area compresses. The cue ball continues to move forward as the compression starts to move the object ball. As the object ball picks up speed, at some point it is moving just as fast as the cue ball.





This "equal-speed" point happens to be at the time of maximum compression, when the flat spot is largest, and when half of the energy of the shot is stored in the compression of the surfaces of the two balls. After this point, the compression releases like a spring, and that stored energy is put back into the object ball. If no energy is lost in the balls — and they are really quite close to perfect — the push-off of the object ball from the compression will stop the cue ball dead.

I bet you didn't know that all of that goes on when you shoot a stop.

How big is the flat spot? You can find it yourself by putting a piece of carbon paper (if you can find one) in front of an object ball and then noticing the size of the mark the cue ball leaves. It also works to use freshly waxed balls or balls with a wet film from your condensed breath. The result is that for a hard shot, the flat spot is a quarter-inch or six millimeters in diameter. How much did the surface of each ball compress during such a collision? Simple geometry says about 0.3mm or one hundredth of an inch. That's about the thickness of three sheets of typing paper.

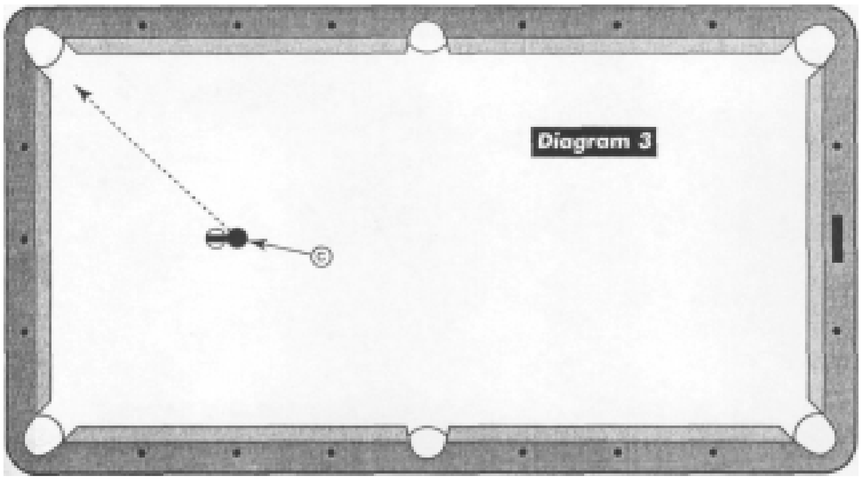
How long does this collision take? It can be measured just from the size of the flat spot and the speed of the cue ball. It has also been measured directly by Wayland Marlow in an experiment described in his book, *The Physics of Pocket Billiards.* He measured the time the balls were in contact by having them make an electrical connection (that's the hard part) and then measuring the time of the contact (that's the easy part). His typical result was 200 millionths of a second. It is important that this time is much longer than the time it takes for sound to travel from one side of the ball to the other, or just as for the cue stick, the "echo" of energy from the far side of the ball couldn't take part in the shot.

The ball-ball collision is hiding a further subtlety that can be important in play. As the balls compress together, the interaction is not like a simple spring. Instead it is a sort of compound spring, because as the flat spot gets larger, more and more surface area gets involved. This means that the "spring" gets "harder" the more the balls are compressed. For a normal cue-ball-to-object-ball collision, this makes little difference, but when another object ball is behind the first object ball, as when two balls are spotted on the long string, the exact nature of the collision becomes important.

What happens during the shot is illustrated in **Diagram** 2. This is for balls that are perfectly springy (elastic) and the prediction is a little surprising. Theory says that for perfect balls, the cue ball is expected to bounce back from the two-ball collision with some speed. For the "progressive spring," the speed back is expected to be about 8% of the incoming speed. If the balls behaved like regular springs, governed by Hooke's Law, the bounce-back is expected to be twice that large.

Why do three balls behave so much differently that two? It turns out that for just two balls, the Laws of Conservation of Energy and Momentum forbid anything except the stop shot from happening. If three balls are involved, there are many outcomes that satisfy the two Laws, and which outcome we see can only be predicted if we include the exact details of how the balls interact. The three balls must be all touching at the same time during the collision.

So, if the cue ball is expected to bounce back, how come we never see it do so? (Draw doesn't count.) The answer seems to be that enough energy is lost in the collision that the cue ball is fully stopped, but doesn't get enough push-back from the springs to get negative motion. You might try the experiment yourself with an old set of balls, and then some right out of the box.



What happens to the middle ball is the useful part. It is going forward some, which is contrary to the usual teaching of "stop-shot physics." To make a shot from this for the double-spot shot, as shown in **Diagram 3,** place the cue ball a little off line, and hit the front ball full. It will get some speed to the side, but will also have some speed forward, due to the "three-ball effect." If you have the right small angle, you can make the front ball in the corner pocket. This kind of shot has been described here before as the "10-times-fuller" system. The factor of 10 can be predicted from the physics. Also, since the compression length is so small (less than 0.3mm), the shot is greatly changed if the two balls that are supposed to be frozen are even the thickness of a dollar bill apart.

The compound spring law that governs spheres in collision is called Hertz' law, and it was discovered by the same physicist who discovered radio waves and whose name you see in "megahertz" and such terms. The details of the law are an active area of research, and not just on the pool table. It turns out that industrial processes which transport beads or pellets of material have lots of ball-ball collisions going on, and Hertz' Law is needed to understand them. If you have access to the Web, enter "Hertz contact law sphere" in a good search engine, and you should get plenty of equations.

If you've waded through all of this rather technical stuff, you deserve a reward. Here's an old puzzle, slightly reworded. If you get the correct answer and are the entry chosen by our autocratic judge — me — you'll get a one-year subscription to this magazine. It's better to send e-mail to jewett@sfbilliards.com, but real mail sent to *BD* in my attention is OK.

In 1887, the bright, young Mr. Hertz was walking down a street in Berlin when he heard an unfamiliar clicking sound coming from a tavern. Entering, he saw a teacher and a pupil and three ivory billiard balls. Hertz had heard of this "billiards" but had never seen a table before. The teacher shot a simple carom — with the click that had attracted Hertz — and explained the 90-degree initial carom angle. Hertz piped up, "But the angle between the paths of the balls must be less than 90 degrees." How did he know? For extra credit, what was his mistake?