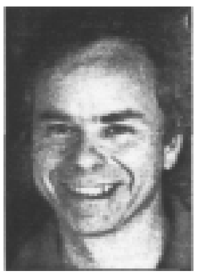
Pythagoras & Pool Perpendiculars

by BOB JEWETT



EVERY BOOK ABOUT pool that gets far enough into the subject to discuss combinations or position play says that the "kiss angle," or the angle between two colliding balls, is 90 degrees or a right angle. None of them tells you why. The explanation takes only a little physics plus your favorite theorem from high school geometry — the one by Pythagoras. Simple extensions of the analysis show how the kiss angle changes if throw or imperfect balls are included in the calculations.

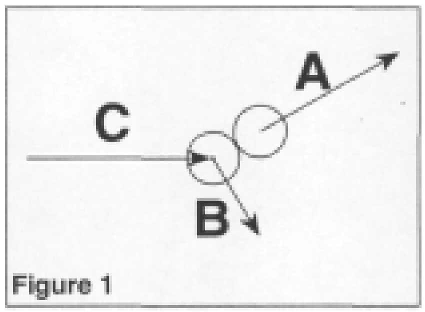


Figure 1

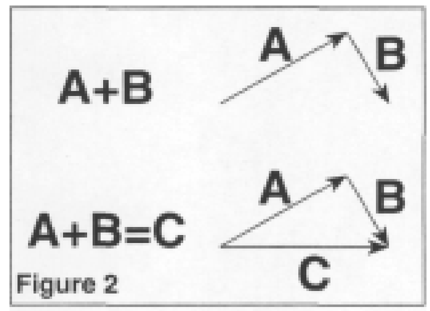


Figure 2

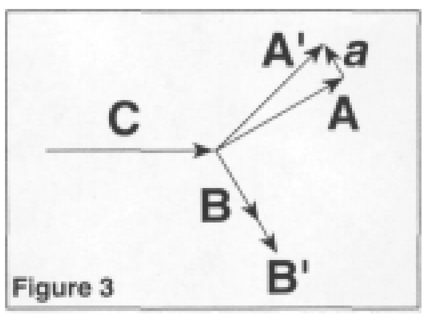


Figure 3

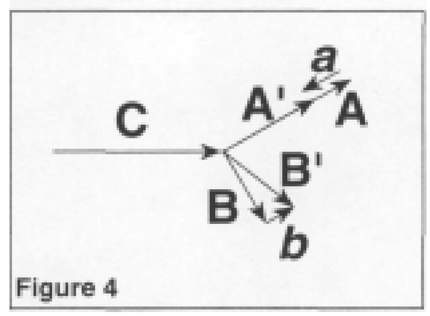


Figure 4

Figure 1 diagrams a simple collision. The cue ball (or it might be an object ball) arrives along path C, sending the target ball along path A. The cue ball leaves along path B. For the time being, we will assume that there is no throw, so path A is along the line joining the centers of the two balls at the instant of the collision.

How do we know the angle between A and B is 90 degrees?

The solution is found by applying basic laws of physics: momentum and energy are neither created nor destroyed during the collision, although they are both transferred from one ball to another.

The momentum of an object is a tricky concept in physics because it has both a size and a direction. The amount of momentum increases with the speed and the weight of the object. The direction of momentum is simply the direction which the object is moving. In Figure 1, the lengths of the arrows represent the speeds of the balls, so the arrows show both the size and direction of the momentum of each ball.

All the momentum before the collision is carried by the cue ball. (The object ball has no momentum since it is standing still and has no speed.) This starting momentum is represented by arrow C. After the collision, some momentum has been transferred to the object ball (arrow A) and the cue ball has kept some (arrow B). The total momentum after the collision. A+ B, is the same amount of momentum as before the balls contacted each other, C, so A+B=C.

This equation doesn't take into account the directions involved. In physics, this is done by "vectors," which for our purposes means we need to add the A and B arrows graphically by putting them head to tail and keeping their original directions as shown in Figure 2. If you go in the A direction for the A length and then in the B direction for the B length, you get to the same spot as going in the C direction for the C length. You may want to check that if you go in the order B, A you still get to the same place.

The main point of the equation is that A, B and C form a triangle since the two paths from the left end of C go to the same point at the right end of C.

The amount of energy in a moving object increases with the weight of the object, just like momentum. Unlike momentum, energy increases as the square of the object's velocity. A physicist calculates an object's energy with the following equation: E = *'A* x mass x velocity^2. So the initial energy is *'A* x m x C^2. After the collision, the energy is shared by the two balls, (*'A* x m x A^2) + (*'A* x m x B^2). Since energy can be neither created or destroyed, that is the second law of thermodynamics, and assuming that no energy goes into heat or sound, the energy before and after the collision must be equal:

$$(\tfrac{1}{2} \times m \times A^2) + (\tfrac{1}{2} \times m \times B^2) = \tfrac{1}{2} \times m \times C^2$$

Using algebra, that equation can be reorganized as:

$$\tfrac{1}{2} \times m \times (A^2 + B^2) = \tfrac{1}{2} \times m \times C^2$$

Since *'A* and m (the mass of one ball) are on both sides of the equal sign, algebra allows us to drop them from the equation altogether:

$$A^2 + B^2 = C^2$$

To summarize what we know so far, the

"velocity vectors" of the balls form a triangle . Due to conservation of energy, the three sides of the triangle have the relationship $A^2 + B^2 = C^2$. This is just the requirement of the Pythagorean Theorem, which states that any right triangle has $A^2 + B^2 = C^2$ if the 90-degree angle is between sides A and B. *QED,* as Mrs. Morgan use to say in geometry class.

Now that we have the basic result, it's time to examine some of the variations that can arise with use of English. Suppose there is some throw in the shot, for example if the cue ball has right (counterclockwise) side-spin. There will be a small change, arrow a, to the A direction, as shown in Figure 3, which results in A'. It's important to note that the direction of a is perpendicular to A because that's the direction of the rubbing of the side-spin. In order to conserve momentum, B must change by an equal and opposite amount, resulting in B'.

For this outside English, the angle between A and B is widened by just the throw angle between A and A'. Furthermore, the cue ball speed is slightly increased, from B to B', which can be estimated as about 7% for maximum English and a cut shot of about 45 degrees.

Conversely, if left English were used, the angle between the cue and the object ball would close up some and the cue ball would leave the collision with slightly lower speed than before.

What happens if the balls are different weights? The problem becomes much more difficult to analyze, but the basic result is that for a heavy cue ball, the kiss angle is less than 90 degrees, while for a light cue ball it will be wider.

What happens if the balls are a little inelastic or "dead"? In that case, some energy of motion is lost during the collision as heat energy, not as much speed is transferred to the object ball, and A is shortened to A' as shown in Figure 4. Momentum is conserved even when energy is changing "type," so B must change by an equal and opposite amount to B' and the angle closes up to less than 90 degrees. This effect is very noticeable with ivory balls, which are not as springy as plastic.

This column has mostly dealt with the "why" rather than the "how" of the kiss angle. The cue ball path is so severely restricted by the physics of the collision that there is little room for adjustment if the object ball is to be driven to a precise target. Of course, after the collision, the cue ball can be maneuvered with draw and follow, but in that first instant Newton and Pythagoras are in control.